Compressive sensing and Tomography

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Imaging and Measurements in Biomedical Engineering
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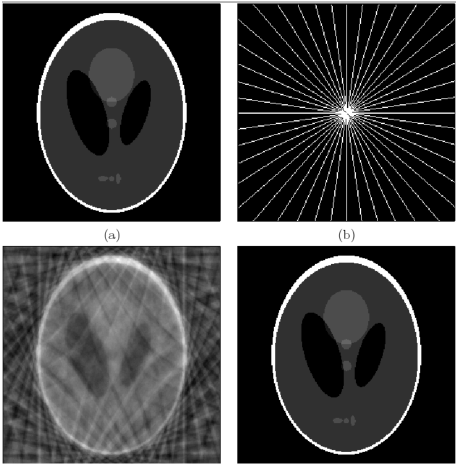
1. The experiment of Candès, Romberg and Tao
2. Information Location
3. Uniqueness
4. Actual tomography



Numerical experiment found in the article from Candès, Romberg, and Tao.

Problem solved (1)

Image reconstruction through the minimization of the total variation, under the constraint that the Fourier transform of the image is known on a subset (sub-sampling).

Principle

The problem has an unique solution, and this is the original image.

Reproducibility

Written as a feasibility problem :

Problem

$$\textit{Find } x \in S = S_1 \cap S_2, \tag{1}$$

with the following constraint sets :

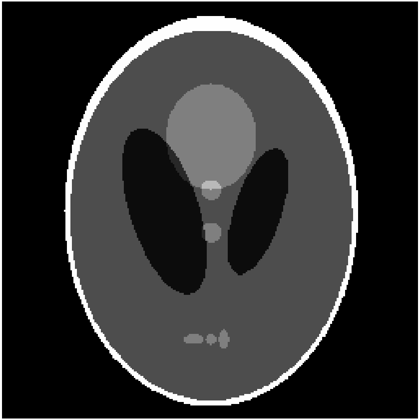
1. *Fourier transform known on the mask K :*

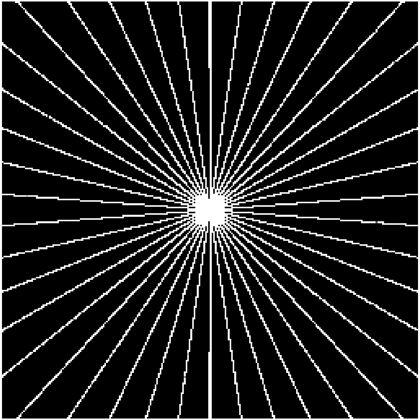
$$S_1 = \left\{ x \in \mathcal{H} \mid \widehat{x} 1_K = a 1_K \right\}. \tag{2}$$

2. *Bounded total variation :*

$$S_2 = \left\{ x \in \mathcal{H} \mid \mathrm{tv}(x) \leq \mathrm{tv}(\overline{x}) \right\}. \tag{3}$$

Data

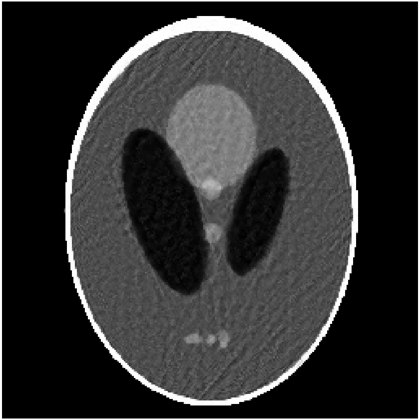


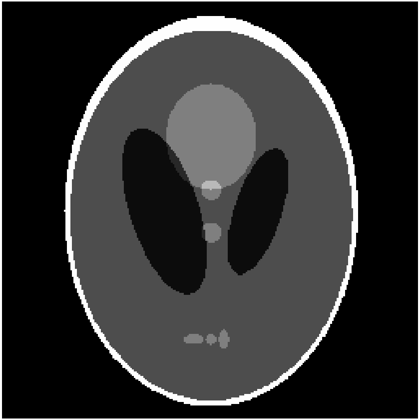


22 views, no noise.

Results

Without and with the constraint on total variation.

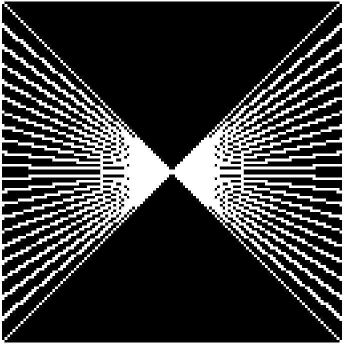


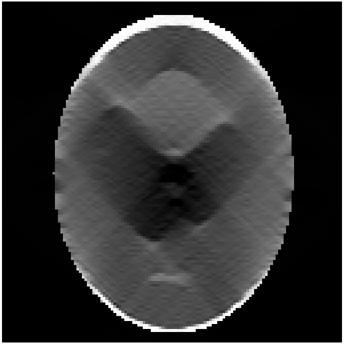


Importance of the information location

Nothing is assumed about the location of the information in the Fourier domain.

Reconstruction with limited angle





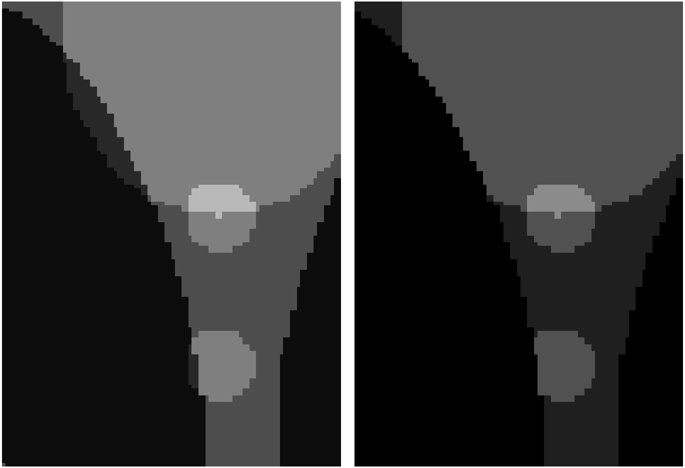
Example with 22 views uniformly spread along 90 degrees, without any noise. The constraints used are the data from measurements, and various *a priori* on : the pixel range($x \in [0;1]^N$), the total variation, and the support.

Mask layout

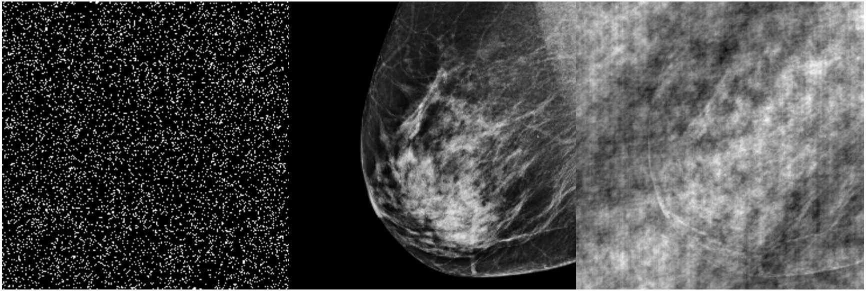
We use a random mask with the same “sparsity” (11%). This first example is a simple case, with broad uniform zones.

Mask layout

The geometry is not reconstructed neither.



A realistic case (mammography)



Uniqueness in tomography

A uniqueness result in tomography is really surprising

The problem of uniqueness

	1	1	1	1	1	1
1	0	1	1	0	0.5	0.5
1	1	0	0	1	0.5	0.5

- An infinity of solutions.
- The solution to the problem (1) is not the original image.

Information about the mean value

Information on the mean value of the image.

1. The (discrete) total variation is invariant through translation by vector $[1, \cdots, 1]^\top$ (uniform image).
2. With a sparse random mask, there is a huge probability that the Fourier coefficients will not contain this information.

$\longrightarrow$ This is a second conceptual difficulty for uniqueness.

Actual tomography

Tomography is not about sub-sampling in the Fourier domain

- Data acquisition noise.
- The projection slice theorem can not be applied as easily in the discrete case. $\longrightarrow$ at least the quantization noise needs to be considered.
- Not all images have broad uniform zones.

See also : Herman, G. T., Davidi, R. : On image reconstruction from a small number of projections. *Inverse Problems,* to appear (2008).

Conclusions

- The experiment from Candès, Romberg and Tao must be understood as a reconstruction from a sub-sampling of the Fourier domain, without noise and using a mask adapted to simple signals.
- All the code to reproduce those experiments is available on my webpage.